

The Phase Diagram of the $U(2) \times U(2)$ Sigma Model *

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We study the phase diagram of the $U(2) \times U(2)$ scalar model in $d = 4$ dimensions. We find that the phase transition is of first order in most of the parameter space. The theory can still be relevant to continuum physics (as an effective theory) provided the transition is sufficiently weakly first order. This places restrictions on the allowed coupling constants.

1. The $U(N) \times U(N)$ model

We consider a scalar field theory described by the action

$$S(\phi) = \int d^4x \left(\frac{1}{2} \text{Tr}(\partial_\mu \phi^\dagger \partial_\mu \phi) + \frac{1}{2} m^2 \text{Tr}(\phi^\dagger \phi) + \lambda_1 (\text{Tr} \phi^\dagger \phi)^2 + \lambda_2 \text{Tr}(\phi^\dagger \phi)^2 \right), \quad (1)$$

in Euclidean space, where $\phi(x)$ is a complex $N \times N$ matrix. The action (1) is invariant under a $U(N)_L \times U(N)_R$ (to be taken $N = 2$) symmetry, under the global symmetry transformation $\phi \rightarrow L\phi R^\dagger$, where L, R are $U(N)$ matrices. This model has been considered as a low-energy effective theory to describe the strong-coupling extended technicolor models and top-condensate models of electroweak symmetry breaking [1].

This model, in contrast to the $O(N)$ model, is known to possess for $\lambda_2 \neq 0$ a first order phase transition whose strength varies in the (λ_1, λ_2) space. This is due to the well-known Coleman-Weinberg instability [2], and it places restrictions on the allowed parameter space of couplings. As one adjusts m^2 past a critical value, m_c^2 , the vacuum expectation value (v.e.v.) v jumps discontinuously from zero in the unbroken phase to some finite nonzero value in the broken phase. Then, if the model is to be a valid low-energy effective theory, relevant to continuum physics, the couplings (λ_1, λ_2) , should belong to a region where the phase transition is sufficiently weak first order. It is only then that v can be small compared to the cut-off Λ .

The model (1) has been studied in perturbation theory, in terms of the effective potential and

the renormalization group (RG) in ref. [3] and in the language of RG flows and its associated fixed points [4,5] in [1,6]. A preliminary investigation on the lattice was undertaken in ref. [7]. We present here a more complete investigation of the phase structure of the $U(2) \times U(2)$ model, by performing Monte Carlo simulations of the lattice regularized version of the action (1) above.

2. Perturbation theory

In the sequel we take $N = 2$, so the action (1) depends on eight degrees of freedom. More details about the model can be found in [7,8]. If $\lambda_2 = 0$, then the symmetry is enhanced to $O(8)$ ($O(2N^2)$ for general N). The pattern of symmetry breaking depends on the sign of λ_2 . If $\lambda_2 > 0$ then the breaking occurs according to

$$U(2)_L \times U(2)_R \rightarrow U(2)_V \quad (2)$$

resulting in four Goldstone bosons, while if $\lambda_2 < 0$, the symmetry breaking pattern is that of

$$U(2)_L \times U(2)_R \rightarrow U(1)^3 \quad (3)$$

resulting in five Goldstone bosons.

Fig. (1) displays the RG flows, at one-loop (solid lines) and two-loop (dash lines) level, within renormalized perturbation theory. Then, starting from bare couplings $(\lambda_1(\Lambda), \lambda_2(\Lambda))$, if $\lambda_2 \neq 0$ all RG flows in the infrared intersect the “stability line” [4], eventually becoming runaway trajectories. The phase transition is then of first order [5]. For λ_2 small, though, the flow is rather slow and even though the “stability line” is crossed, this happens after many decades of running; the phase transition is, in this case, weakly

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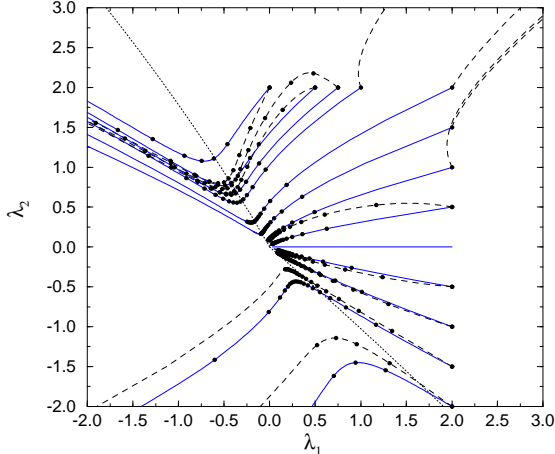


Figure 1. Perturbative RG trajectories starting from bare couplings $(\lambda_1(\Lambda), \lambda_2(\Lambda))$ along the lines $\lambda_1 = 2$ or $\lambda_2 = 2$. The solid lines (dash lines) correspond to one-loop (two-loop) trajectories while the stability line is indicated as a dotted line. Indicatively, the dots along a trajectory represent the evolution of couplings after running by a factor of e down to the infrared.

first order, with $v \ll \Lambda$. Along the $\lambda_2 = 0$ axis the transition is known to be second order, and the well known triviality analysis [9] applies. The two-loop corrections seem to improve the hierarchy Λ/v , as found also in [6]. In particular, there exists a region with $\lambda_1, \lambda_2 > 0$ where the flow is towards larger values and it appears that it never crosses the stability line. However, this only hints upon the breakdown of perturbation theory and a nonperturbative analysis is called for. It is also essential that no other fixed point, unreachable in perturbation theory, exists. Should one be present, the RG trajectories would be distorted and there could be regions where the transition is second order. We found no evidence of such a fixed point.

On the lattice, the physical parameter controlling the running of the couplings and hence the size of the corrections is the correlation length ξ of the system. Then one expects that $\Lambda/v \sim \xi^\beta$, where β is the appropriate critical exponent: a large hierarchy will only be possible if ξ is big, or equivalently, that the transition is weakly first order or second order.

Table 1

Estimates of the jump in the order parameter v^2 evaluated at the critical point m_c , and of the correlation length estimated by $\xi \simeq L^*/2$, where L^* is the smallest lattice where coexistence was found, or from the effective potential.

(λ_1, λ_2)	m_c^2	v^2	ξ
(0.5, -0.45)	-0.772	0.83	7
(-0.22, 0.5)	-0.91	2.10	3
(0, 0.5)	-2.42	0.5	40
(-3.97, 8)	-1.30	10-20	< 2
(-14.97, 30)	-1.50	20-40	< 2
(0, 8)	-24.75	0.15	6
(0, 16)	-43.98	0.15	6
(0, 30)	-77.0	0.19	6
(8, 8)	-63.8	0.11	6
(8, 16)	-82.5	0.16	6
(8, 30)	-114.5	0.17	6

3. Monte Carlo Results

Table I shows all the points in the (bare) coupling constant space at which we performed Monte Carlo simulations. We used two different programs checked against each other: one based on a simple one-hit Metropolis algorithm with a uniform step tuned so that the acceptance rate is about 60% and the other based on the hybrid algorithm (with or without Fourier acceleration). In this latter method, two parameters have to be chosen, namely the number of leap-frog steps and the step size. This allows better control of the autocorrelation time. We found optimal CPU performance for 5-8 leap-frog steps before each Metropolis test. The hybrid one performed clearly better.

We used as an order parameter (following [7]) the expectation value of the $U(N) \times U(N)$ invariant operator $O = \text{Tr} \bar{\phi}^\dagger \bar{\phi}$ which corresponds to the susceptibility, where $\bar{\phi}$ is the lattice average of each field component. $\langle O \rangle$ is proportional to v^2 in the broken phase and zero in the unbroken phase, modulo finite size corrections.

We used lattices of sizes ranging from $L^4 = 4^4$ to $L^4 = 14^4$. In order to obtain information about the order of the transition, at each given (λ_1, λ_2) we searched for hysteresis effects in the measurement of the order parameter by performing ther-

mal cycles in the relevant parameter, m^2 , across the critical region. Strong hysteresis loops is an indication of a strong first order transition.

On smaller lattices, $(4^4, 6^4, 8^4)$, the critical region was identified by searching for a double-peak signal in the histogram distribution of $\text{Tr}(\phi\phi^\dagger)$. We then moved to bigger lattices $(10^4, 12^4, 14^4)$ to look for coexistence. Along the process of increasing the lattice size we eventually begin to see metastability at some size L^* . We estimate then the correlation length to be $\xi \sim L^*/2$. Crude as this procedure may seem, it is physically meaningful and it agrees, where comparison is possible, with the effective potential.

Our results are as follows. All points close to the stability line exhibited marked hysteresis loops and hence show strong first order transitions, becoming stronger as we move up along the stability line. For such couplings, $\xi \simeq 1$ so the cut-off effects are big and the connection to continuum physics questionable. In the weak coupling region ($\lambda_1, \lambda_2 < 1$), we also computed the one-loop bare effective potential [7] and found that it agreed with the numerical data within 10 – 30%. Next we investigated couplings along the λ_2 axis. Typically, runs on $4^4, 6^4$ lattices did not show any hysteresis effects. However we found clear sign of the existence of two minima on $10^4 - 14^4$ lattices. Fig. (2) displays tunneling between two phases for the point (0, 8) on a 10^4 lattice. For the same point, clear signal was found of two coexistent minima on the 12^4 lattice, but no tunneling was observed. The transition becomes stronger with increasing λ_2 . For points deep in the $\lambda_1 > 0, \lambda_2 > 0$ region, we were able to observe coexistence of phases, but only on $12^4, 14^4$ lattices. The transition is always clearly first order, but characterized by correlation lengths, as expected, larger than those obtained close to the stability line. For couplings close to the λ_1 axis, our numerical results are in agreement with the expectation of the RG for a weak first order transition. In this case, though, it is difficult to distinguish a weak first order from a second order transition.

Our results are summarized in Table (1). From these, one can see that in most of parameter space the v.e.v., $v(\Lambda)$, is typically only one order of magnitude smaller than the cut-off. Our results

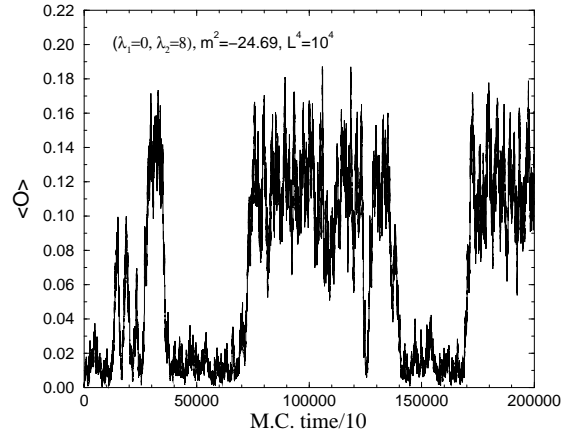


Figure 2. Order parameter time history (over 2×10^6 hybrid sweeps) for the point (0, 8) on a 10^4 lattice at $m^2 = -24.69$.

are consistent with the standard perturbative picture of first order phase transitions and the absence of any nontrivial fixed point. The hierarchy Λ/v is not “tunable” by m^2 as in the $O(N)$ model, but rather depends on λ_2 . Phenomenologically viable models must lead to couplings with small λ_2 in order to support a large hierarchy.

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